REFERENCES

[1] P. D. Wendt, E. J. Coyle, and N. C. Gallagher, "Stack filter," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 898–911, 1986.

[2] J. H. Lin and E. J. Coyle, "Minimum mean absolute error estimation over the class of generalized stack filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, pp. 663–678, 1990.

[3] E. J. Coyle and J. H. Lin, "Stack filters and the mean absolute error criterion," *IEEE Trans. Acount., Speech, Signal Processing*, vol. 36, pp. 1244–1254, 1988.

[4] M. Gabbouj and E. J. Coyle, "On the LP which finds a MMAE stack filter," *IEEE Trans. Signal Processing*, vol. 39, pp. 2419–2424, 1991.

[5] J. H. Lin, T. M. Sellke, and E. J. Coyle, "Adaptive stack filtering under the mean absolute error criterion," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, pp. 938–954, June 1990.

[6] J. H. Lin and Y. T. Kim, "Fast algorithm for training stack filters," *IEEE Trans. Signal Processing*, vol. 42, pp. 772–781, Apr. 1994.

[7] J. B. Bednar and T. L. Watt, "Alpha-trimmed means and their relation to median filters," *IEEE Trans. Acoust. Speech Signal Processing*, vol. ASSP-32, pp. 145–153, 1984.

[8] F. Pasian, "Sorting algorithms for filters based on ordered statistics: Performance considerations," *Signal Processing*, vol. 14, pp. 287–293, 1988.

[9] C. C. Han and K. C. Fan, "A greedy and branch and bound searching algorithm for finding the optimal morphological erosion filter on binary images," *IEEE Signal Processing Lett.* vol. 1, pp. 41–44, 1994.

[10] ——, "A greedy and $A^\star$ searching based approach to the finding of optimal binary morphological filter," *SPIE Visual Commun. Image Processing*, vol. 2308, pp. 110–118, 1994.

[11] C. C. Han, "Design and searching of optimal morphological filter on binary mesh pattern images," Ph.D. dissertation, Nat. Cent. Univ., Taiwan, R.O.C., 1994.

[12] P. Maragos and R. S. Schafer, "Morphological filters. Part II: Their relations to median, order-statistic, and stack filters," *IEEE Trans. Acoust. Speech Signal Processing*, vol. ASSP-35, pp. 1170–1184, 1987.

[13] P. Maragos, "A representation theory for morphological image and signal processing," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 11, pp. 586–599, June 1989.

[14] S. Agaian, J. Astola, K. Egiazarian, and P. Kuosmanen, "Decompositional methods for stack filtering using Fibonacci p-codes," *Signal Processing*, vol. 41, pp. 101–110, 1995.

Scale Periodicity and its Sampling Theorem

Hari Sundaram, S. D. Joshi, and R. K. P Bhatt

***Abstract*— The scale transform is a new representation for signals, offering perspective that is different from the Fourier transform. In this correspondence, we introduce the notion of a scale periodic function. These functions are then represented through the discrete scale series. We also define the notion of a strictly scale-limited signal. Analogous to the Shannon interpolation formula, we show that such signals can be exactly reconstructed from exponentially spaced samples of the signal in the time domain. As an interesting, practical application, we show how properties unique to the scale transform make it very useful in computing depth maps of a scene.**

Manuscript received December 13, 1994; revised October 14, 1996. The associate editor coordinating the review of this paper and approving it for publication was Dr. Jelena Kovačević.

H. Sundaram is with the Department of Computer Science, Columbia University, New York, NY 10027 USA (e-mail: hari@cs.columbia.edu).

S. D. Joshi and R. K. P. Bhatt are with the Department of Electrical Engineering, Indian Institute of Technology, Delhi, India (e-mail: sdjoshi@ee.iitd.ernet.in; rkpb@ee.iitd.ernet.in).

Publisher Item Identifier S 1053-587X(97)04964-7.

I. INTRODUCTION

The Fourier transform is one of the most widely used tools in signal processing. There are several major reasons for this, the most important one being that exponentials are eigenfunctions of linear time-invariant systems. That the discrete Fourier transform can be efficiently computed using the FFT algorithm has also contributed to its popularity.

A transform closely related to the Fourier transform is the scale transform. The scale transform was introduced by Cohen in an operator theoretic framework [1]. In it, he defines a scale operator and the corresponding eigenfunctions defining the scale transform. Uncertainty relations and notions of bandwidth were also developed.

The scale transform has been shown to be related to the Fourier transform via an unitary transformation [5]. There are several reasons why the scale transform should merit attention on its own. Many practical applications, especially those in computer vision, are shift variant by nature (e.g., depth from defocus). In such cases, one cannot use the Fourier transform. Since tools for manipulating signals in the Fourier domain are widely available, shift-variant systems are often assumed to be piecewise linear shift invariant. In some shift-variant problems, the scale transform proves to be useful (see Section V, where depth maps are computed using the scale transform). The scale transform also provides a distinctly different signal representation from the one provided by the Fourier transform. This representation, in terms of *scalings* of a basis function, often gets overlooked due to the preeminence of linear time invariant systems in signal processing.

The Fourier series have long been used for representing periodic signals. The Fourier transform developed from the Fourier series out of the need to represent aperiodic signals. The scale transform, having being developed in an abstract operator theoretic framework, lacks a sense of historical development. There is, in fact, a very natural progression toward the scale transform via a class of functions known as *scale periodic* functions. In this paper, we develop notions of a scale periodic function and the discrete scale series that help represent such functions. In order for the scale transform to be useful in signal processing applications, one must be able to reconstruct an arbitrary aperiodic signal from its samples in the time domain. Analogous to Shannon's sampling theorem, we show that a scale-limited signal can be exactly reconstructed from samples that are exponentially spaced along the time axis.

The scale transform is briefly reviewed in the next section. The generic problem where the scale transform proves useful is also introduced Section II. The main results of this paper are presented in Sections III and IV. Section III introduces scale periodic functions and the discrete scale series. In Section IV, we develop the interpolating formula for scale-limited signals. Section V discusses a new technique for the computation of depth maps.

II. THE SCALE TRANSFORM

A signal can be represented in a variety of ways, depending on the attribute of the signal we wish to emphasize. A transform defined in [1] that is related via a unitary transformation to the Fourier transform is the scale transform

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(c)\, \frac{e^{jc \ln t}}{\sqrt{t}}\, dc \tag{1}$$

$$F(c) = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} f(t)\, \frac{e^{-jc \ln t}}{\sqrt{t}}\, dt. \tag{2}$$

TABLE I
PROPERTIES OF THE SCALE TRANSFORM SIGNAL AND TRANSFORM

Signal	Transform
$x(t)$	$X(c)$
$y(t)$	$Y(c)$
$A\,x(t)+B\,y(t)$	$A\,X(c)+B\,Y(c)$
$t^n f(t)$	$F(c+nj)$
$\sqrt{\frac{1}{2\pi}}\int_0^\infty \frac{1}{\tau}f_1(\frac{1}{\tau})f_2(t\tau)d\tau$	$F_1(c)F_2(c)$

where $F(c)$ is defined to be the scale transform of $f(t)$. Note the intervals of integration in (2). The function $f(t)$ is defined on $(0, \infty)$. Hence, for arbitrary signals, the positive and the negative parts of the signal have to be treated separately. The function $\gamma(c, t) = e^{jc \ln t}/\sqrt{2\pi t}$; $t \geq 0$ is actually the eigenfunction of the scale operator C [1], which is defined as

$$C = \frac{1}{2j}\left(\frac{d}{dt}t + t\frac{d}{dt}\right). \tag{3}$$

Consider the property of the scale transform

$$f(\beta t) \Leftrightarrow \frac{1}{\sqrt{\beta}}F(c)e^{-jc \ln \beta} \tag{4}$$

where $F(c)$ is the scale transform of $f(t)$, and β is a constant. What is really interesting about (4) is that the support of the scale transform of $f(\beta t)$ is the same as that of $f(t)$. This implies that the scale transform can used to extract a constant from the argument of a function whose scale transform is *a priori* known. In the case of the Fourier transform, as is well known, the size of the support of the Fourier transform of $f(\beta t)$ is different from that of $f(t)$. That is

$$f(\beta t) \Leftrightarrow \frac{1}{|\beta|}F\left(\frac{\omega}{\beta}\right) \tag{5}$$

where $F(\omega)$ is the Fourier transform of $f(t)$. Some of the other important properties of the scale transform are shown in Table I.

Consider the generic problem

$$f_1(t) = g\{tu_1[\alpha(t)]\} \tag{6}$$
$$f_2(t) = g\{tu_2[\alpha(t)]\} \tag{7}$$

where $f_1(t)$ and $f_2(t)$ are measured quantities, $g(t)$, $u_1(t)$, $u_2(t)$ are *a priori* known, while $\alpha(t)$ is the quantity to be estimated. This problem is difficult to solve for $\alpha(t)$ using the Fourier transform. This generic problem is found in many computer vision applications, notably in the computation of depth maps of a scene. An approximate solution for $\alpha(t)$ can be found by using the scale transform, as illustrated in Section V.

From the discussions on scale periodic signals and the sampling theorem, it will be realized that the scale transform actually represents a change in *paradigm* of signal representation. It forces us to think of signals as being representable by *scalings* of a basis function rather than in terms of shifts (as is true in the case of the representation of bandlimited signals).

III. SCALE PERIODICITY AND THE SCALE SERIES

A function $g(t)$ is defined to be periodic in time if it satisfies $g(t) = g(t+T)\,\forall t$, where T is the fundamental period of the function. Fourier analysis began as an attempt to represent such functions using sinusoids. The scale transform was developed in an abstract operator theoretic framework [1], and hence, its usefulness in signal processing applications is not readily apparent. In order to appreciate the utility of the scale transform, it is important to understand its origins. A class of signals known as *scale periodic* signals leads naturally to the development of the scale transform.

TABLE II
PROPERTIES OF THE SCALE SERIES PERIODIC SIGNAL SERIES

Periodic Signal	Series
$x(t)$ period τ	a_k
$y(t)$ period τ	b_k
$A\,x(t)+B\,y(t)$	$A\,a_k+B\,b_k$
$\sqrt{\alpha}x(\alpha\,x), \alpha>0$	$e^{jkC_o \ln \alpha}\,a_k$
$x^*(t)$	a_k^*
$e^{jmC_o \ln t}x(t)$	a_{k-m}
$\int_1^\tau \frac{1}{u}x\left(\frac{1}{u}\right)y(tu)du$	$\ln\tau\, a_k b_k$
$\sqrt{t}x(t)y(t)$	$\sum_{m=-\infty}^{\infty} a_m b_{k-m}$
$t^{(\alpha-1)/2}x(t^\alpha)$ period $\tau^{1/\alpha}$	a_k

Definition: Let $\mathbf{S}$ be the intersection of the function spaces $L^2(a, b)\,\forall b/a = \tau$ for a fixed $\tau (a > 0\, b > 0)$. A function $f(t) \in \mathbf{S}$ is said to be scale periodic with period τ if it satisfies

$$\sqrt{\tau}f(t\tau) = f(t)\,\forall t > 0; \qquad \tau > 0. \tag{8}$$

C_o is defined to be the fundamental scale associated with this function $f(t)$, where $C_o \ln \tau = 2\pi$. In addition, note that this space $\mathbf{S}$ is trivially non empty for $1/\sqrt{t} \in \mathbf{S}$.

In the set of eigenfunctions $\{(1/\sqrt{2\pi})\,(e^{jC \ln t}/\sqrt{t})\}$, only eigenfunctions for which $C = kC_o$ are scale periodic with period τ. In the discussion that follows, we will assume $\tau > 1$ for the sake of definitiveness, although all the results are valid for the case $0 < \tau < 1$.

A representation of $x(t) \in \mathbf{S}$, a scale periodic function with period τ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \frac{e^{jkC_o \ln t}}{\sqrt{t}}; \qquad a \leq t < b \tag{9}$$

$$a_k = \frac{1}{\ln \tau}\int_a^b x(t)\frac{e^{jkC_o \ln t}}{\sqrt{t}}dt; \qquad \frac{b}{a} = \tau \tag{10}$$

where the equality in (9) is under the standard norm in L^2 spaces.

These are the analysis (10) and the synthesis (9) equations for the scale periodic function $x(t)$. It is interesting to note that the eigenfunctions $e^{jkC_o \ln t}/\sqrt{t}$ and $e^{jnC_o \ln t}/\sqrt{t}$ are orthogonal over any interval (a, b), where $b/a = \tau$, $C_o \ln \tau = 2\pi$. Each eigenfunction is scale periodic with period τ. This should be contrasted with the Fourier case, where the eigenfunctions $e^{jk\omega_o t}$ and $e^{jn\omega_o t}$ are orthogonal over any interval (a, b), where $b - a = T$, $\omega_o T = 2\pi$. Each eigenfunction is periodic in time with period T.

Similar to Parseval's relation for the Fourier series, we can show the identity

$$\int_{\tau^{-1/2}}^{\tau^{1/2}} |x(t)|^2\, dt = \ln \tau \sum_{k=-\infty}^{\infty} |a_k|^2. \tag{11}$$

Other properties of the discrete scale series are summarized in Table II, all of which can be easily verified. It can also be shown that in the limit $\tau \to \infty$, the discrete scale series tends to the scale transform. The discrete scale series can be proven to be complete. The proof is similar in nature to the proof of completeness of the Fourier series [4].

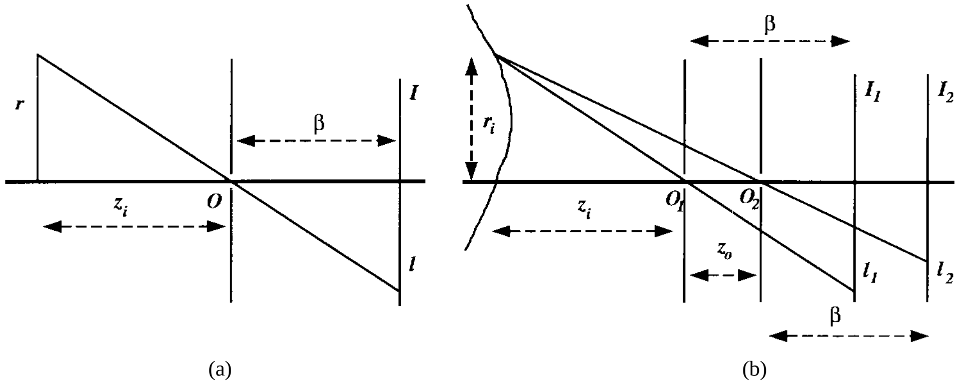

Fig. 1. (a) Simple imaging geometry showing the scaling in a pinhole camera. The image sensor I is at a fixed distance β from the pinhole. The scaling can be shown to be $l = \beta r / z_i$. (b) Scene is captured on two images I_1 and I_2 at different locations. After one image of the scene has been captured in I_1, the pinhole camera is moved backward along the optical axis by a distance of z_o, and the scene is imaged in I_2. The depth of the patch at (r_i, z_i) is determined by applying the scale transform to corresponding scaled patches in I_1 and I_2.

IV. The Sampling Theorem

Theorem: A function $f(t) \in L^2(R)$ scale limited to C_o can be exactly reconstructed (in the L^2 sense) from its samples in the time domain if the samples are spaced exponentially along the time axis as in $\{\tau^n\}_{n=-\infty}^{\infty}$, where $\tau = e^{\pi/C_o}$.

The motivation behind the development of the sampling theorem is to determine conditions under which an aperiodic signal can be exactly reconstructed from its samples in the time domain. An idea similar to bandwidth is needed here.

Definition: A function is defined to be strictly scale limited to scale C_o if the support of $F(c)$, which is the scale transform of $f(t)$, is limited to C_o, i.e., $F(c) \equiv 0 \, \forall \, c \notin (-C_o, C_o)$.

Consider the function $\gamma(t)$ defined as

$$\gamma(t) = \sqrt{\frac{2}{\pi}} \, \frac{\sin (C_o \ln t)}{\sqrt{t} \ln t}. \tag{12}$$

The scale transform of $\sqrt{\tau^m}\gamma(t\tau^m)$, where $C_o \ln \tau = \pi$, can be shown to be

$$\Gamma(c) = \begin{cases} e^{-jcm \ln \tau} & |c| \le C_o \\ 0 & \text{elsewhere.} \end{cases} \tag{13}$$

Consider a signal $f(t)$, whose scale transform $F(c)$ is strictly scale limited to C_o. Due to the Fourier series representation, we have

$$F(c) = \frac{1}{\sqrt{2\pi}} \sum_{m=-\infty}^{\infty} a_m e^{jcm \ln \tau} \tag{14}$$

$$a_m = \frac{1}{\sqrt{2\pi}} \int_{-C_o}^{C_o} F(c) e^{-jcm \ln \tau} \, dc. \tag{15}$$

Using the definition of the scale transform for $f(t)$

$$f(t) = \int_{-C_o}^{C_o} \frac{F(c)}{\sqrt{2\pi t}} \, e^{jc \ln t} \, dc \tag{16}$$

we get

$$a_m = \ln \tau \sqrt{\tau^{-m}} f(\tau^{-m}). \tag{17}$$

Now, using (14) and (17), we can easily derive the interpolation formula for scale-limited signals.

$$f(t) = \int_{-C_o}^{C_o} \frac{F(c)}{\sqrt{2\pi t}} \, e^{jc \ln t} \, dc \tag{18}$$

$$= \frac{1}{\sqrt{2\pi t}} \sum_{m=-\infty}^{\infty} a_m \int_{-C_o}^{C_o} e^{jc \ln (t\tau^m)} \, dc \tag{19}$$

$$= \sum_{m=-\infty}^{\infty} a_m \sqrt{\frac{2}{\pi t}} \, \frac{\sin [C_o \ln (t\tau^m)]}{\ln (t\tau^m)}. \tag{20}$$

Now, substituting for a_m from (17), we get

$$f(t) = \sqrt{\frac{2}{\pi}} \ln \tau \sum_{m=-\infty}^{\infty} f(\tau^m) \, \frac{\sin [C_o \ln (t\tau^{-m})]}{\sqrt{t\tau^{-m}} \ln (t\tau^{-m})}. \tag{21}$$

This interpolation shares similarities to the Shannon interpolating formula with interesting differences. The function is represented as a weighted sum of the *dilations* of a basis function $\gamma(t)$. The dilation factor τ is related to the maximum scale of the signal C_o as $2C_o \ln \tau = 2\pi$. In addiiton, note that the weights are the original function $f(t)$ sampled exponentially rather than linearly as in the Shannon formula.

V. The Scale Transform and Depth Maps

One of the most extensively studied problems in computer vision is the problem of depth map computation of an arbitrary scene. There are many passive[1] techniques that are being used at present, notably, depth from focus, depth from defocus, and stereo. A new passive technique for computation of depth maps of a scene via the scale transform is demonstrated.

Consider the situation shown in Fig. 1(a). The apparatus used is a simple pinhole camera coupled with an imaging sensor (e.g., a ccd array) at a fixed distance of β from the pinhole. Hence, the captured image is merely a scaled, inverted version of the scene. The scaling for each point in the image depends on the depth of the scene point. The relation between the scene point and its imaged point is shown to be

$$l = \frac{\beta r}{z_i} \tag{22}$$

where l and r are the image and scene coordinates, respectively, measured with respect to the optical axis (i.e., the axis passing through the pinhole), and z_i is the depth of the scene point.

In Fig. 1(b), the scheme for depth recovery, using two images of the scene, is illustrated. The example shown is the simple 1-D[2] profile, and the case of the general 2-D surface is a straightforward extension. Let the profile depicted be the function $g(r, z)$. Consider $f(r)$ (a small patch defined locally)[3] located at an arbitrary point (r_i, z_i). We can assume that the depth in this small patch is essentially a constant. The pinhole camera is now used to capture the scene. The

[1] A passive technique as opposed to an active one does *not* interact with the environment. An example of an active depth technique is the laser range finder.

[2] One could also consider a solid of revolution, in which case, a 1-D analysis suffices.

[3] The coordinate system for each patch is local to it.

patch $f(r)$ located at (r_i, z_i) is imaged as a small scaled patch $f_1(l)$ centered at (l_1, β). Now, the pinhole camera is moved backward by a distance z_o along the optical axis, and the scene is captured again. The same patch at (r_i, z_i) is now imaged as another scaled patch $f_2(l)$ centered at (l_2, β). Using (22), we can easily show that

$$f_1(l) = f\left(\frac{lz_i}{\beta}\right) \tag{23}$$

$$f_2(l) = f\left[\frac{l(z_i + z_o)}{\beta}\right]. \tag{24}$$

We now use the property that the scale transform of $f(\alpha t)$ is $F(c)e^{-jc \ln \alpha}/\sqrt{\alpha}$. Taking the scale transforms of (23) and (24), we get

$$F_1(c) = \sqrt{\frac{\beta}{z_i}}\, F(c)e^{-jc \ln (z_i/\beta)} \tag{25}$$

$$F_2(c) = \sqrt{\frac{\beta}{z_i + z_o}}\, F(c)e^{-jc \ln [(z_i+z_o)/\beta]}. \tag{26}$$

Now, by taking the modulus and dividing, we can determine the depth z_i of the patch centered at (r_i, z_i) as

$$\sqrt{\frac{z_i + z_o}{z_i}} = \frac{|F_1(c)|}{|F_2(c)|} \tag{27}$$

$$z_i = \frac{z_o |F_1(c)|^2}{|F_1(c)|^2 - |F_2(c)|^2}. \tag{28}$$

Note that before we can use this technique, one must solve the correspondence problem, i.e., determining the two patches f_1 and f_2 that are related to each other. This correspondence problem could be modeled as a joint optimization problem, solving for correspondence and z_i simultaneously. This correspondence problem appears in many computer vision problems, including stereo and motion estimation.

Instead of modeling the depth of the patch located at (r_i, z_i) to be a constant, one can also assume the depth to be varying linearly across the patch, as in $z = a_i r + b_i$. One can then set up equations to solve for the unknown constants a_i and b_i. In this case, we would need to take three images instead of two.

The above algorithm is a simple illustration of a problem where it is natural to use the scale transform rather than the Fourier transform.

VI. Conclusions

The scale transform is an important tool that has yet to be fully exploited in signal processing applications. It must be noted, however, that the scale transform supplements the Fourier transform, rather than replacing it. The example of depth map computation shows the utility of the scale transform in certain shift-variant problems. It is important to provide a context through which the scale transform can be intuitively understood. We have done so through a notion of the scale periodic function and the discrete scale series. In order that arbitrary signals be analyzed using the scale transform, we have defined the notion of a strictly scale-limited signal. We have shown that strictly scale-limited signals can be exactly reconstructed using an interpolation formula that is similar in flavor to the Shannon interpolation formula.

Acknowledgment

One of the authors (H. Sundaram) would like to thank S. K. Nayar for valuable discussions. He would also like to thank K. Dana and S. Baker for help in reviewing this correspondence.

References

[1] L. Cohen, "The scale representation," *IEEE Trans. Signal Processing,* vol. 41, pp. 3275–3292, Dec. 1993.

[2] S. Mallat, "Multifrequency channel decompositions of images and wavelet models," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 37, pp. 2091–2110, 1989.

[3] D. Marr and E. Hildreth, "Theory of edge detection," in *Proc. Royal Soc. Lond.,* vol. B-207, pp. 187–217, 1980.

[4] A. Zygmond, *Trignometric Series Vol. I.* Cambridge, U.K.: Cambridge Univ. Press, 1959.

[5] R. G. Baraniuk and D. L. Jones, "Unitary equivalence: A new twist on signal processing," *IEEE Trans. Signal Processing,* vol. 43, pp. 2269–2282, Oct. 1995.

On the Efficient Prediction of Fractal Signals

Yifeng Zhou, Patrick C. Yip, and Henry Leung

***Abstract*—A novel prediction scheme for self-affine fractal signals is presented. The signal is modeled by self-affine linear mappings, whose contraction factors are assumed to follow an auto-regressive (AR) process. In this way, the highly nonlinear time evolution of the fractal signal is captured by the linear AR process of the contraction factors, thereby exploiting the simplicity and ease of computation inherent in the AR model. An adaptive version of the proposed scheme is applied in simulations using the Weierstrass–Mandelbrot cosine fractal, as well as, in practice, using real radar sea clutter data.**

I. Introduction

Much interest has been shown in examining the fractal behavior of natural phenomena since the publication of the momentous work by Mandelbrot [1]. From the artificial depiction of natural sceneries in motion pictures to the study of stock market fluctuations, fractals have found interesting applications. Some phenomena previously thought to be random have been found to be deterministic but fractal. In engineering, an outstanding example is the discovery that radar returns from sea clutter are not random but form a series or signal with a fractal dimension [2]. The implication that sea clutter radar returns are amenable to deterministic modeling has led to new and successful target detection schemes in sea clutter environments [2]–[4]. However, much of the modeling done is based on nonlinear approaches that suffer from a heavy computational load.

Iterated function system (IFS) theory has recently received considerable attention [5]–[7]. IFS's have the ability of producing complicated functions and fractals with any noninteger dimension [7]. The IFS theory has been successfully used to model the discrete-time sequences in the context of data compression [9]–[13]. In IFS applications, linear fractal interpolation is of special interest because it is computationally less demanding than those nonlinear fractal interpolation procedures. In linear fractal interpolation, the fractal

Manuscript received July 31, 1995; revised February 18 1997. The associate editor coordinating the review of this paper and approving it for publication was Dr. Farokh Marvasti.

Y. Zhou is with Telexis Corporation, Ottawa, Ont., Canada K1V 7P2.

P. C. Yip is with Communications Research Laboratory, McMaster University, Hamilton, Ont., Canada L8S 4K1.

H. Leung is with Surface Radar Section, Defense Research Establishment Ottawa, Ottawa, Ont., Canada K1A OK2.

Publisher Item Identifier S 1053-587X(97)04965-9.